

► **Problem 9.2-26(b)(d)**

Let $V = \{1, 2, 3, \dots, n\}$.

- (a) How many graphs are there with vertex set V ? **Ans:** $2^{\binom{n}{2}}$.
- (b) How many of the graph in (a) contain the triangle 123?
- (c) What is the total number of triangles in all the graphs with vertex set V ? **Ans:** $\binom{n}{3}2^{\binom{n}{2}-3}$.
- (d) On average, how many triangles does a graph on n labeled vertices contain?

Solution. (b) In this case, three of edges are determined to be in the graph, but we still have a free choice from among the remaining $\binom{n}{2} - 3$. so the number is $2^{\binom{n}{2}-3}$.

$$(d) \text{ average} = \frac{\text{total number of triangles in all graphs}}{\text{total number of graphs}} = \frac{\binom{n}{3}2^{\binom{n}{2}-3}}{2^{\binom{n}{2}}} = \frac{\binom{n}{3}}{8} \quad \square$$