▶ **Exercise 10-1**

Let $G = (V, E)$ be a Hamiltonian graph. Prove that $k(G - S) \leqslant |S|$ for every nonempty proper subset $S$ of vertices of $G$.

**Proof.** Let $S$ be a non-empty proper subset of $V$. Suppose that $k(G - S) = p$ and that $G_1, G_2, \ldots, G_p$ are the components of $G - S$. Since $G$ is Hamiltonian, $G$ contains a hamiltonian cycle. Whenever $C$ encounters a vertex of $G_i$ for the last time ($1 \leqslant i \leqslant p$), the next vertex of $C$ must belong to $S$. This implies that $S$ must contain at least $p$ vertices, that is, $k(G - S) = p \leqslant |S|$. □