► **Exercise 10-3**

Let u and v be non-adjacent vertices in a graph G of order n such that $\deg u + \deg v \geqslant n$. Prove that $G + uv$ is Hamiltonian if and only if G is Hamiltonian. (Hint: use Ore's Theorem. See Problem 10.2-16 in textbook.)

Proof. If G is a Hamiltonian graph, then certainly $G + uv$ is Hamiltonian for any two non-adjacent vertices u and v of G. Thus we need only verify the converse.

Let $G + uv$ be a Hamiltonian graph for two non-adjacent vertices u and v of a graph G, and assume, to the contrary, that G is not Hamiltonian. This implies that every Hamiltonian cycle in $G + uv$ must contain the edge uv and so G contains a Hamiltonian path from u to v. Since $\deg u + \deg v \geqslant n$, the proof of Ore's Theorem tells us that G contains a Hamiltonian cycle. This is a contradiction. $\qquad \square$