— Date: 2009/10/1 —

▶ **Exercise 9-1**

Let $S = \{2, 3, 4, 7, 11, 13\}$. Draw the graph G whose vertex set is S and such that (i, j) is an edge of G for $i, j \in S$ if $i + j \in S$ or $|i - j| \in S$.

▶ **Exercise 9-2**

For a graph $G = (V, E)$, the number of vertices of G (i.e., $|V|$) and the number of edges of G (i.e., $|E|$) are called *order* and *size* of G, respectively. The degree of every vertex of a graph G of order 25 and size 62 is 3, 4, 5, or 6. There are two vertices of degree 4 and 11 vertices of degree 6. How many vertices of G have degree 5?

▶ **Exercise 9-3**

Prove that if a graph of order $3n$ ($n \geqslant 1$) havs n vertices of each of the degrees $n - 1$, n, and $n + 1$, then n is even.

▶ **Exercise 9-4**

The degree of every vertex of a graph G of order $2n + 1 \geqslant 5$ is either $n + 1$ or $n + 2$. Prove that G contains at least $n + 1$ vertices of degree $n + 2$ or at least $n + 2$ vertices of degree $n + 1$.

▶ **Exercise 9-5**

If the sequence $x, 7, 7, 5, 5, 4, 3, 2$ is graphical, then what are the possible values of x ($0 \leqslant x \leqslant 7$)?

— Date: 2009/10/9 —

To do the exercises, you need the following definitions.

Definition 1 The *complement* of a graph $G = (V, E)$, denoted by $\overline{G}$, is the graph whose vertex set is the same as G and such that for each pair of vertices $u, v \in V$, uv is an edge of $\overline{G}$ if and only if uv is not an edge of G. A graph G is *self-complementary* if $G \cong \overline{G}$.

Definition 2 Let $G = (V, E)$ be a graph of order n and let k be an integer such that $0 \leqslant k \leqslant n-1$. If the degree $\deg v = k$ for every vertex $v \in V$, then G is called a *k-reguar graph* or a *regular graph of degree k*. Moreover, we say that G is a *regular graph* if G is k-regular for some integer k.

▶ **Exercise 9-6**

Prove that if P and Q are two longest paths in a connected graph, then P and Q have at least one vertex in common.

▶ **Exercise 9-7**

Let G be a graph of order 5 or more. Prove that at most one of G and $\overline{G}$ is bipartite.

▶ **Exercise 9-8**

Let G be a self-complementary graph of order $n = 4k$, where $k \geqslant 1$. Let $U = \{v : \deg v \leqslant n/2\}$ and $W = \{v : \deg v \geqslant n/2\}$. Prove that if $|U| = |W|$, then G contains no vertex v such that $\deg v = n/2$.

▶ **Exercise 9-9**

Let G_1, G_2, and G_3 be pairwise disjoint connected regular graphs and let $G = G_1 + (G_2 + G_3)$ be the graph obtained from G_1, G_2, and G_3 by adding edges between every two vertices belonging to two of G_1, G_2, and G_3. Prove taht if G_1 and $\overline{G_1}$ are Eulerian, but G_2 and G_3 are not Eulerian, then G is Eulerian.

▶ **Exercise 9-10**

How many non-isomorphic graphs have the degree sequence s: 6, 6, 6, 6, 6, 6, 6, 6, 6?

— Date: 2009/10/24 —

To do the exercises, you need the following definitions.

Definition 3 A graph G that is not connected is called *disconnected*. A connected subgraph of G that is not a proper subgraph of any other connected subgraph of G is a *component* of G. Let $k(G)$ denote the number of components of G.

Definition 4 The *subdivision graph* of a graph G, denoted by $S(G)$, is the graph obtained from G by deleting every edge uv of G and replacing it by a vertex w of degree 2 that is joined to u and v.

▶ **Exercise 10-1**

Let $G = (V, E)$ be a Hamiltonian graph. Prove that $k(G - S) \leqslant |S|$ for every nonempty proper subset S of vertices of G.

▶ **Exercise 10-2**

Is it true that if $S(G)$ is Hamiltonian, then G is Eulerian?

▶ **Exercise 10-3**

Let u and v be non-adjacent vertices in a graph G of order n such that $\deg u + \deg v \geqslant n$. Prove that $G + uv$ is Hamiltonian if and only if G is Hamiltonian. (Hint: use Ore's Theorem. See Problem 10.2-16 in textbook.)

▶ **Exercise 10-4**

For a graph $G = (V, E)$ of order $n \geqslant 3$, then the graph $G(3)$ is obtained from G by adding a new vertex v_S for each 3-element subset S of V and joining v_S to each vertex in S. Find all such graphs G for which $G(3)$ is Hamiltonian.

▶ **Exercise 10-5**

Determine whether the following graph is Hamiltonian.