▶ **Exercise 9-2**

For a graph $G = (V, E)$, the number of vertices of G (i.e., $|V|$) and the number of edges of G (i.e., $|E|$) are called *order* and *size* of G, respectively. The degree of every vertex of a graph G of order 25 and size 62 is 3, 4, 5, or 6. There are two vertices of degree 4 and 11 vertices of degree 6. How many vertices of G have degree 5?

Solution. Since $|E| = 62$, the total degree of the graph G is $62 \cdot 2 = 124$. Let x be the number of vertices of degree 3, and let y be the number of vertices of degree 5. From the fact that there are two vertices of degree 4 and 11 vertices of degree 6, we have

$$\begin{cases} x + y = 25 - (2 + 11) = 12 \\ 3x + 5y = 124 - (2 \cdot 4 + 11 \cdot 6) = 50 \end{cases}$$

Therefore, we can solve the equation to obtain $x = 5$ and $y = 7$, which means that there are 7 vertices of degree 5. □