

► **Exercise 9-3**

Prove that if a graph of order $3n$ ($n \geq 1$) has n vertices of each of the degrees $n - 1$, n , and $n + 1$, then n is even.

Proof. Since the graph has n vertices of each of the degrees $n - 1$, n , and $n + 1$, the total degree of the graph is $n \cdot (n - 1) + n \cdot n + n \cdot (n + 1) = 3n^2$, which is an even number by the Euler formula. Obviously, this implies that n is even. \square