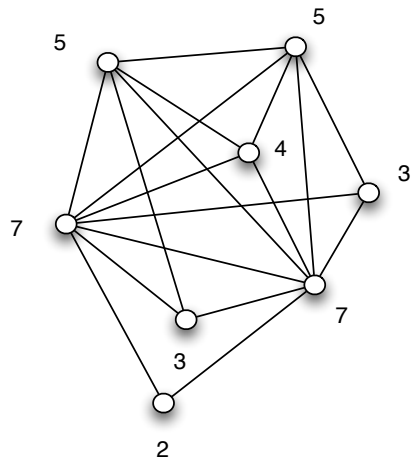
▶ **Exercise 9-5**

If the sequence $x, 7, 7, 5, 5, 4, 3, 2$ is graphical, then what are the possible values of x $(0 \leqslant x \leqslant 7)$?

Solution. Since if a sequence is graphical then the number of odd integers must be even, this shows that the possible values of x are 1, 3, 5, or 7. However, since there are 8 vertices in the graph and at least two of the vertices have degree 7, this implies that $x \neq 1$. Also, since there is a vertex of degree 2, it implies that $x \neq 7$. We now check the degree sequences $7, 7, 5, 5, 4, 3, 3, 2$ and $7, 7, 5, 5, 5, 4, 3, 2$ for $x = 3$ and $x = 5$, respectively, as follows. From the checking results, we conclude that $x = 3$ and $x = 5$ are possible.

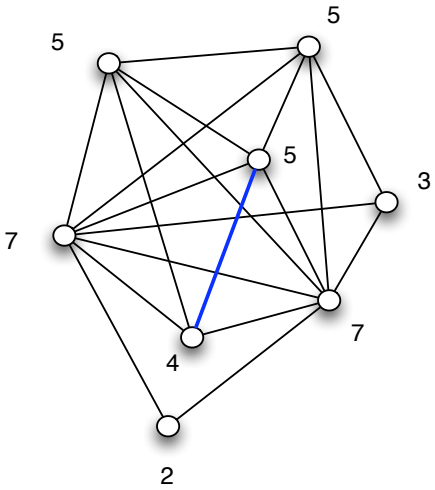
For $x = 3$,

$$
\begin{aligned}
& 7, 7, 5, 5, 4, 3, 3, 2 \\
\rightarrow\ & 6, 4, 4, 3, 2, 2, 1 \\
\rightarrow\ & 3, 3, 2, 1, 1, 0 \\
\rightarrow\ & 2, 1, 0, 1, 0 \\
\text{sort} \rightarrow\ & 2, 1, 1, 0, 0 \\
\rightarrow\ & 0, 0, 0, 0
\end{aligned}
$$



For $x = 5$,

$$
\begin{aligned}
& 7, 7, 5, 5, 5, 4, 3, 2 \\
\rightarrow\ & 6, 4, 4, 4, 3, 2, 1 \\
\rightarrow\ & 3, 3, 3, 2, 1, 0 \\
\rightarrow\ & 2, 2, 1, 1, 0 \\
\rightarrow\ & 1, 0, 1, 0
\end{aligned}
$$



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