▶ **Exercise 9-7**

Let G be a graph of order 5 or more. Prove that at most one of G and $\overline{G}$ is bipartite.

Proof. If G is not bipartite, then we have the desired result. Thus, we may assume that G is bipartite. Let V_1 and V_2 be two bipartition sets of G. Since the order of G is at least 5, at least one of V_1 and V_2 contains 3 or more vertices, say $|V_1| = p \geqslant 3$. Since the subgraph of $\overline{G}$ induced by V_1 is the complete graph K_p and $p \geqslant 3$, it follows that $\overline{G}$ contains a triangle. Since every bipartite graph has no odd cycles, $\overline{G}$ is not bipartite. □