▶ **Exercise 9-9**

Let G_1, G_2, and G_3 be pairwise disjoint connected regular graphs and let $G = G_1 + (G_2 + G_3)$ be the graph obtained from G_1, G_2, and G_3 by adding edges between every two vertices belonging to two of G_1, G_2, and G_3. Prove taht if G_1 and $\overline{G_1}$ are Eulerian, but G_2 and G_3 are not Eulerian, then G is Eulerian.

Proof. Let G_i be r_i-regular of order n_i $(i = 1, 2, 3)$. Since G_1 is Eulerian, r_1 is even. Since $\overline{G_1}$ is Eulerian, $n_1 - r_1 - 1$ is even. Thus, n_1 is odd. Since G_2 is not Eulerian, r_2 is odd and so n_2 is even. Similarly, r_3 is odd and n_3 is even. Observe that

(1) Every vertex of G_1 in G has degree $r_1 + n_2 + n_3$, which is even.

(2) Every vertex of G_2 in G has degree $r_2 + n_1 + n_3$, which is even.

(3) Every vertex of G_3 in G has degree $r_3 + n_1 + n_2$, which is even.

Since G is connected and every vertex has even degree, G is Eulerian. □