▶ **Problem DS-02-02** Suppose $T_1(N) = O(f(N))$ and $T_2(N) = O(f(N))$. Which of the following are true?

(a) $T_1(N) + T_2(N) = O(f(N))$.

(b) $T_1(N) - T_2(N) = o(f(N))$.

(c) $\frac{T_1(N)}{T_2(N)} = O(1)$.

(d) $T_1(N) = O(T_2(N))$.

Solution.

(a) **True**. By Rule 1 (in Page 16 of textbook), we have

$$T_1(N) + T_2(N) = \max\{O(f(N)), O(f(N))\} = O(f(N)).$$

(b) **False**. For example, we consider $T_1(N) = 2N = O(N)$ and $T_2(N) = N = O(N)$. In this case, $f(N) = N$ and $T_1(N) - T_2(N) = 2N - N = N = \Theta(f(N))$. However, by definition $T_1(N) - T_2(N) = o(f(N))$ implies $T_1(N) - T_2(N) \neq \Theta(f(N))$.

(b) **False**. For example, we consider $T_1(N) = N^2 = O(N^2)$ and $T_2(N) = N = O(N^2)$. In this case, $f(N) = N^2$ and $\frac{T_1(N)}{T_2(N)} = \frac{N^2}{N} = N \neq O(1)$.

(b) **False**. For example, we consider $T_1(N) = N^2 = O(N^2)$ and $T_2(N) = N = O(N^2)$. In this case, $f(N) = N^2$ and $T_1(N) = N^2 \neq O(N) = O(T_2(N))$. □